INFLUENCE OF LONGITUDINAL TEMPERATURE GRADIENT ON LIGHT PROPAGATION IN LENS-LIKE MEDIUM

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It is shown that the combined longitudinal and transverse temperature gradient can be used to obtain new types of focusing properties in lens-like media.

The influence of the longitudinal temperature gradients is sometimes important in actual thermal-hydrodynamic gas lenses [1]. To date, however, published studies have concentrated on the influence of the longitudinal temperature gradient in gas lenses [2, 3]. The combined influence of the longitudinal and transverse gradients on the ray trajectory $y = y(x)$ has not received sufficient attention. Here we shall consider the propagation of a light ray in a flat lens-like gas medium, where the index of refraction depends on the longitudinal and transverse coordinates. The focusing and defocusing regions are found as a function of certain relationships between the gradient of the index of refraction along both the x and y axes.

Formulation of Problem. Basic Equations. Assumptions. The fundamental equation describing light propagation in a medium with index of refraction $n(x, y)$ takes the following form in the geometric-optics approximation:

$$y'' - \frac{\partial \ln n}{\partial y}(1 + y'^2) + \frac{\partial \ln n}{\partial x}(y' + y'^3) = 0. \tag{1}$$

To this equation we must add the boundary conditions

$$y(x=0) = y_0, \quad y'(x=0) = y_0'. \tag{2}$$

It is of interest to consider the following type of relationship between the index of refraction and the coordinates, which is often found in applied problems:

$$n(x, y) = f(y)e^{cx}, \tag{3}$$

where c is a constant, and $f(y)$ is an arbitrary function.

It is impossible to solve the analytic equation (1) in the general case with the index of refraction determined by (3). Thus we shall solve it under two physically valid simplifications, and shall consider the following two corresponding cases:

1) we let $y'^2 \ll 1$; this means that the ray trajectories are slowly varying functions;

2) y'^2 is not small, but the longitudinal variations in the index of refraction are moderate; we can then assume that c is small, and seek a solution to (1) in the form of a series in the small parameter c.

Thus in the first case, Eq. (1) takes the form

$$y'' + cy' - \varphi(y) = 0, \tag{4}$$

where

$$\varphi = \frac{1}{n}\frac{\partial n}{\partial y} = \frac{1}{f}\frac{df}{dy}.$$

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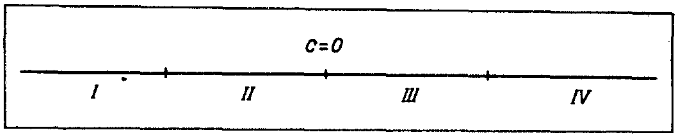

Fig. 1. Regions of aperiodic (I) and periodic (II) defocusing, and periodic (III) and aperiodic (IV) focusing.

Let us look at the distribution of the index of refraction,

$$n = n_0 e^{cx - \gamma y^\nu}, \tag{5}$$

which is determined by the following temperature profile:

$$T = n_0^{-1}(n_0 - 1)\, T_0 \exp(\gamma y^\nu - cx).$$

When $\nu = 1$, we write the solution of (4) as

$$y = Be^{-cx} - \frac{\gamma}{c}\, x + \left(\frac{A}{c} + \frac{\gamma}{c^2}\right),$$

where

$$B = -\left(\frac{y_0'}{c} + \frac{\gamma}{c}\right); \quad A = cy_0 + y_0'.$$

It is not difficult to see that when x is sufficiently large, and the dimensions of the transition region are large, regardless of the initial values, all trajectories will asymptotically approach the line $y = -\gamma x/c$. Physically, this means that the lens-like medium with the index of refraction (5) will focus an incoming beam, and rotate it through the angle $\arctan(-\gamma/c)$.

When $\nu = 2$, we have the following three cases:

1. $c^2 > 8\gamma$. The solution of (4) takes the form

$$y = c_1 \exp \frac{-c + \sqrt{c^2 - 8\gamma}}{2}\, x + c_2 \exp \frac{-c - \sqrt{c^2 - 8\gamma}}{2}\, x,$$

where

$$c_1 = (c^2 - 8\gamma)^{-1/2}\left(y_0' + y_0 \frac{c + \sqrt{c^2 - 8\gamma}}{c}\right);$$

$$c_2 = -(c^2 - 8\gamma)^{-1/2}\left(y_0' + y_0 \frac{c - \sqrt{c^2 - 8\gamma}}{2}\right).$$

Thus the trajectory found is an aperiodically damped curve.

2. $8\gamma > c^2$. Under such conditions, rays will propagate along trajectories described by the equation

$$y = e^{-\frac{1}{2}cx}\left(c_1 \cos\frac{1}{2}\sqrt{8\gamma - c^2}\, x + c_2 \sin\frac{1}{2}\sqrt{8\gamma - c^2}\, x\right),$$

where

$$c_1 = y_0; \quad c_2 = \frac{2y_0' + cy_0}{\sqrt{8\gamma - c^2}}.$$

It is not difficult to see that when $c = 0$, the trajectories take the form of oscillating curves. Since the index of refraction has a longitudinal gradient, the amplitudes of these oscillations are damped, and the rays approach the axis. The distance at which the rays enter the axial region is determined by the quantity

$$\frac{1}{c} \simeq L.$$

If $c < 0$, the amplitude of the oscillations will increase, and the medium will lose the focusing properties that it had at $c = 0$.

3. $8\gamma = c^2$. The trajectories described by the equation

$$y = e^{-\frac{1}{2}cx}(c_1 x + c_2),$$

where

$$c_1 = \frac{c}{2} y_0 + y_0'; \; c_2 = y_0,$$

aperiodically approach the x axis. For small x, the lens defocuses, forcing the ray away from the x axis, but as x increases, the ray bends toward the axis. The maximum deviation of the ray from the axis occurs at

$$x_{\max} = \frac{2}{\gamma} - \frac{y_0}{y_0' + \frac{c}{2}}$$

and equals

$$y_{\max} = \frac{c + 2y_0}{\gamma} \exp\left[-\frac{c}{a} + \frac{cy_0}{2y_0' + c}\right].$$

Thus the medium acquires focusing properties owing to the presence of the positive gradient of the index of refraction in the x direction.

Let us look at the second case, in which no restrictions are imposed on the trajectories, but we assume that the index of refraction is a weak function of the x coordinate. The small parameter c appears in the problem. We transform Eq. (1) somewhat, taking the new variables $z = dy/dx$, where we let y represent the independent variable.

We then rewrite (1) as

$$z \frac{dz}{dy} = (1 + z^2)[\varphi(y) - cz].$$

Letting $1 + z^2 = u$, we have

$$\frac{du}{dy} = 2u\left[\varphi(y) - c\sqrt{u - 1}\right].$$

We seek a solution to this equation as a series in the parameter c

$$u = u_0 + cu_1 + c^2u_2 + \ldots$$

and we can now write the expressions for the first two approximations,

$$u_0 = e^{A\int \varphi(y')dy'},$$

$$u_1 = -2\int u_0 \sqrt{u_0 - 1}\, dy'.$$

Since

$$\frac{dy}{dx} = z = \sqrt{u_0 + cu_1 - 1},$$

we have the ray trajectory

$$x = B + \int\left\{\sqrt{\exp\left[A\int\varphi(y')\,dy'\right] - 2c\int\exp\left[A\int\varphi dy'\right]\sqrt{\exp\left[A\int\varphi dy'\right] - 1}}\right\}^{-1/2} dy,$$

where A and B are determined from the boundary conditions.

To conclude, let us look at the expressions for the focal distances $f = |y_0/y'(l)|$ [1] for the lens-like media considered:

$$f_1 = \frac{y_0}{Bce^{-cl} - \frac{\gamma}{c}},$$

$$f_2 = \frac{|y_0|}{\left|c_1^* \exp\frac{-c + \sqrt{c^2 - 8\gamma}}{2} l - c_2^* \exp\frac{-c - \sqrt{c^2 - 8\gamma}}{2} l\right|} \quad (c^2 > 8\gamma),$$

$$c_1^* = c_1\left(-c + \sqrt{c^2 - 8\gamma}\right), \; c_2^* = c_2\left(c + \sqrt{c^2 - 8\gamma}\right).$$

Thus the longitudinal index-of-refraction gradient substantially changes the nature of the focusing properties of the lens-like medium. Figure 1 shows the regions of focusing and defocusing for a lens-like medium having an index of refraction of the type (5).

LITERATURE CITED

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